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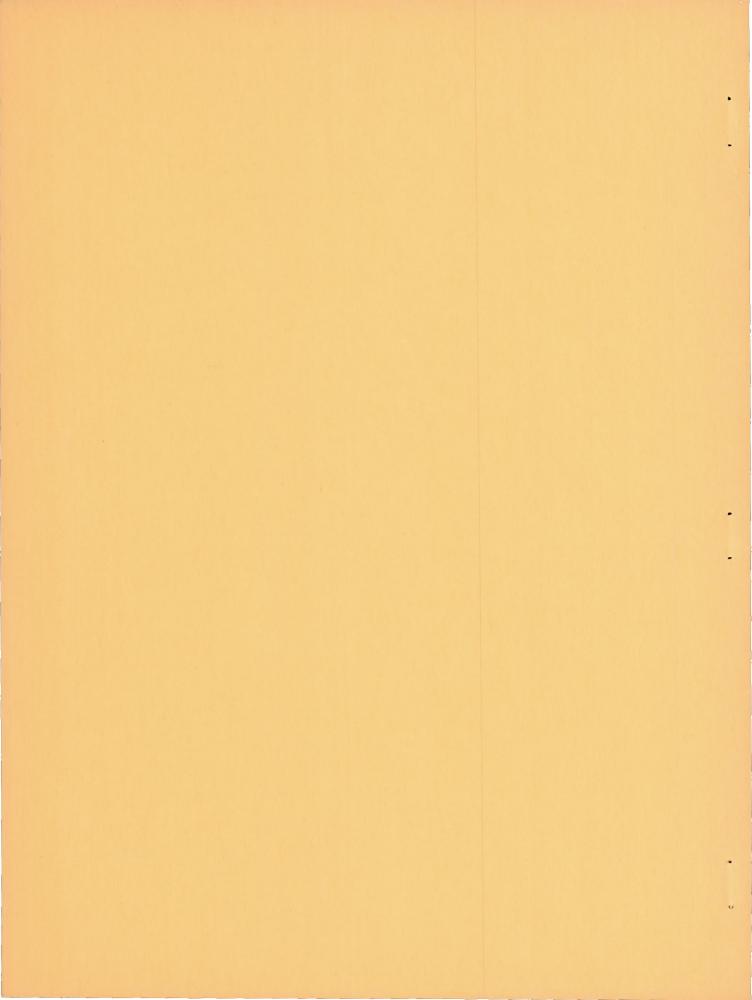
TECHNICAL NOTE 3161

AN INVESTIGATION OF THE USE OF ROCKET-POWERED MODELS FOR

GUST-LOAD STUDIES WITH AN APPLICATION TO A TAILLESS

SWEPT-WING MODEL AT TRANSONIC SPEEDS

By A. James Vitale, H. Press, and C. C. Shufflebarger

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

This report describes the results obtained in an investigation of the
use of rocket-powered models for gust-load studies. A technique for
rocket-powered-model testing in continuous rough air which incorporated
the use of a survey airplane was evolved. In order to test the technique
and at the same time obtain some information on the effects on gust loads
of low damping in pitch, a tailless swept-wing model was flown through
rough air at Mach numbers of 0.80 to 1.00. The results obtained indicate
that the use of rocket-powered models for gust-loads studies is feasible
and practical. The limitations of the technique are discussed and several
suggestions are given for improving the precision of test results.

The tailless model in rough air showed a pronounced pitching motion
at the short-period frequency. The load intensities showed a rapid
increase of roughly 85 percent as the Mach number increased from 0.8 to
1.0. Of this 85-percent increase, 55 percent appeared to be associated
with the rapid decrease of the short-period damping in this Mach number
range. These results were in substantial agreement with the results
obtained from theoretical calculations based on the use of power-spectral
methods of generalized harmonic analysis.

INTRODUCTION

The study of gust loads on airplanes and missiles has been seriously
handicapped by limitations of existing experimental techniques. In gen-
eral, experimental studies of airplane response to gusts have been confined
to the NACA gust tunnel or full-scale flight tests. Reference 1, for
example, describes the gust-tunnel equipment and some tunnel test results.
Reference 2 presents an example of the recent use of flight testing tech-
niques in the study of airplane response to gusts. Existing gust-tunnel

equipment, while useful, has been limited in speed, Mach number, and
Reynolds number, and further, permits determination of only the initial
portion of the model response to a single gust disturbance. Flight tests
have permitted a wider range of testing conditions for continuous gust
disturbances but have been limited by cost, availability of airplanes,
and performance capabilities of existing airplanes. In view of these
limitations of existing experimental techniques for gust-loads studies,
an investigation was undertaken of the feasibility of using rocket-powered
models to extend the range of experimental studies.

As a result of this investigation, a procedure for testing rocket-
powered models in rough air was evolved. This procedure involved the fol-
lowing problems: the choice of a flight time and path to insure flight
through continuous rough air, a means of measuring the intensity of the
rough air, and a model schedule which would provide sufficient record
length at given test conditions. The procedure evolved incorporated the
use of a survey airplane and was applied by the actual firing of a test
model. The configuration used was a tailless swept-wing rocket-powered
model and covered the Mach number range of 0.80 to 1.00. This model was
chosen so that, if the procedure was successful, test data would also be
obtained on a configuration having low damping in pitch. The effect of
low damping in pitch on loads is of considerable interest since recent
theoretical results (ref. 3) had indicated that low damping in pitch could
be expected to give rise to sizeable amplification of loads in rough air.

In the evaluation of the test results and in the associated theoret-
ical analysis presented, extensive use is made of the techniques of gen-
eralized harmonic analysis. The application of these techniques to the
gust-loads case is described in reference 3. These techniques appear
particularly useful in the present application because of the limited
data obtained from rocket-powered-model tests and the unconventional char-
acter of the configuration tested. The present report describes and eval-
uates the test procedures evolved for rocket-powered-model studies in
rough air. The test results for the first model test are evaluated in
order to establish the characteristics of the loads for this configura-
tion and their variation with Mach number and with damping in pitch.
Finally, an analysis aimed at correlating these test results with analyt-
ical results is presented.

SYMBOLS

$c_{1/10}$ cycles to damp to one-tenth amplitude

$\bar{c}$ mean aerodynamic chord, ft

f frequency, cps

g   acceleration due to gravity, 32.2 ft/sec^2

h   altitude, ft

I_y moment of inertia of model about transverse axis, slug-ft^2

$i = \sqrt{-1}$

M   Mach number

p   atmospheric static pressure, lb/sq in.

P   period of motion, sec

Δn normal-acceleration increment, g

S   wing area, sq ft

t   time, sec

T   total time, sec, or temperature of atmosphere, $^\circ$R

$T_{1/2}$ time to damp to one-half amplitude, sec

T( )   frequency-response function

U_{de} derived gust velocity (ref. 4), ft/sec

V   model forward velocity, ft/sec

W   weight, lb

x   distance, ft

σ root-mean-square normal acceleration, g

ρ air density, slugs/cu ft

Φ( )   power-spectral-density function of an arbitrary disturbance
             with respect to f or Ω

Φ_o( )   power-spectral-density function of output or normal acceleration

Φ_i( )   power-spectral-density of input or vertical gust velocity

τ time displacement, sec

Ω reduced frequency, $2\pi f/V$, radians/ft

ω frequency, radians/sec

μ_g mass parameter, $\dfrac{2W}{g \dfrac{dC_L}{d\alpha} S\rho\bar{c}}$

$\dfrac{dC_L}{d\alpha}$ lift-curve slope, per radian

Subscripts:

exp   experimental

calc   calculated

max   maximum

TEST PROCEDURE

Atmospheric Turbulence Conditions Over the Firing Range

The development of a practical rocket-powered-model technique for the study of gust loads requires the assurance that the model will be fired through continuous rough air. Consequently, as a part of this study, airplane survey flights with a suitably instrumented propeller-driven fighter-type airplane were made of the firing range at the Langley Pilotless Aircraft Research Station at Wallops Island, Va. The purpose of these survey flights was to obtain a working basis for selecting suitable days and flight paths for rocket-powered-model tests in rough air. These airplane flights served to establish that, in general, the required conditions of atmospheric turbulence in clear air were most probable with strong westerly (off-shore) winds, particularly post-cold-frontal conditions. For these conditions, the turbulence intensity generally did not vary to any great extent with distance along the firing course for distances up to 5 miles. Also, while the turbulence intensity tended to decrease continuously with altitude from 1,000 to 2,500 feet, the turbulence remained of sufficient intensity over this altitude range to meet the requirements for the present tests. On the basis of these flights, it was decided to schedule the first rocket-powered-model test for a post-cold-frontal weather condition with strong westerly winds and a flight path restricted to altitudes below 2,500 feet.

Model and Instrumentation

A tailless configuration having a $45°$ sweptback wing of aspect ratio 6 and NACA 65A009 airfoil section was used in the present investigation. The principal features of the model are shown in the drawing

of figure 1 and the photographs of figure 2. The wing was mounted on a
fuselage of fineness ratio 10 with the leading edge intersecting the
fuselage contour at the maximum diameter. Listed in table I are the
important physical characteristics of the model. Two flat-plate fins
were used to stabilize the model directionally.

The model instrumentation included a four-channel telemet : trans-
mitting measurements of angle of attack, normal acceleration from accel-
erometers located at the center of gravity and in the nose of the model,
and total pressure. The accelerometers had a natural frequency of about
83 cps and 0.60 damping ratio; thus, the amplitude-response corrections
were negligible up to 50 cycles per second. Tracking instrumentation con-
sisting of a CW Doppler radar set and a modified SCR 584 radar tracking
unit were used to obtain model velocity and position in space.

Model Preflight Tests

Vibration test.- Prior to flight testing, the following model vibra-
tion frequencies were determined by suspending the model from shock cords
attached to forward and rearward sections of the fuselage and mechanically
vibrating the model by a mechanical shaker mounted at the center of gravity:

Wing first bending frequency, cps   33
Wing second bending frequency, cps 144

Longitudinal stability test.- The longitudinal stability character-
istics shown in figure 3 were obtained from previous flight tests in smooth
air of an identical model. These results were obtained by disturbing the
model in pitch with small pulse rockets and analyzing the resulting short-
period free oscillation to obtain the longitudinal stability characteristics.

Airplane Turbulence Survey

A test day was selected (Oct. 3, 1952) on the basis of meteorological
conditions discussed in a previous section of this report. A final check
immediately before model launching was made with the survey airplane
to determine the suitability of turbulence conditions. When the pilot
judged that the intensity of turbulence was satisfactory, the preflight
airplane survey was completed to determine the intensity and variation
with altitude of the turbulence. Flight surveys along the firing course
were made at altitudes of 500, 1,000, 1,500, 2,000, and 2,500 feet with
two runs, 4 to 5 miles in length, made at each altitude (to-sea and
to-land directions). The recorded airspeed-acceleration data at each
altitude were evaluated to determine the gust velocities U_{de} in accord-
ance with the revised gust-load formula of reference 4. Examination of

these data indicated that the turbulence tended to be patchy and varied
somewhat along the flight path. However, no consistent variation with
distance from the shore was discernible. It was therefore assumed that
the flight data were independent of distance from shore, and "Go" and
"Return" runs at each altitude were treated as one set of data in deter-
mining the overall average variation of turbulence intensity with altitude.

The gust velocity equaled or exceeded on the average in 1.0 and
0.1 miles of flight is shown in figure 4 as a function of altitude.
Examination of figure 4 indicates that the turbulence intensity decreased
with altitude above 1,000 feet (in agreement with the pilot's report).
It should be noted, however, that the data of figure 4 represent the
estimated variation of the average turbulence intensity and that con-
siderable variation from the average might be expected for short flight
distances due to the "patchy" character of the turbulence on the test
day.

Model Test

The model was ground launched at an elevation angle of 28° by means
of a 5-inch fin-stabilized booster rocket motor. A photograph of the
model and booster on the launcher is shown in figure 2(b). Following
booster burnout, the model separated from the booster by means of the
difference in drag-weight ratio of the model and booster. After the

model experienced decelerating flight for 10 seconds, a $3\frac{1}{4}$-inch sus-

tainer rocket in the model ignited to accelerate the model to supersonic
speeds for the second time. The variation of Mach number along the model
flight path is illustrated in figure 5(a). By using a long delay in sus-
tainer firing, the record length at each test condition was effectively
doubled since the model covered the Mach number range 0.80 to 1.10 twice
in one flight. As given in table I, there was a small difference in
model weight and moment of inertia for the two portions of the flight,
before and after sustainer rocket burning. Atmospheric conditions at
the time of launching were measured by means of a radiosonde and the
variations of pressure, density, and temperature with altitude are shown
in figure 5(b).

ANALYSIS AND RESULTS

General Considerations

In evaluating the test results and in the associated theoretical
analysis to be presented, use is made of the concepts and techniques of
generalized harmonic analysis. A study of the application of the power-
spectral methods of generalized harmonic analysis to gust-load problems

is presented in reference 3. These methods permit the description of the random atmospheric turbulence disturbance and the associated airplane response in analytic form by means of the so-called "power-spectral-density function." If $y(t)$ is a random disturbance, then the power-spectral-density function $\Phi(f)$ is defined by

$$\Phi(f) = \lim_{T \to \infty} \frac{2}{T} \left| \int_0^T y(t)e^{-2\pi ift}dt \right|^2 \tag{1}$$

where f is frequency in cycles per second, and the vertical bars designate the modulus of the complex quantity. An equivalent and, perhaps, more useful expression for $\Phi(f)$ is given by

$$\Phi(f) = 4 \int_0^\infty R(\tau)\cos 2\pi f\tau \, d\tau \tag{2}$$

where $R(\tau)$ is the autocorrelation function defined by

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T y(t)y(t + \tau)dt$$

A useful property of $\Phi(f)$ is that

$$\int_0^\infty \Phi(f)df = \text{Mean-square disturbance} = \overline{y^2(t)} = \sigma^2 \tag{3}$$

In terms of the power-spectral-density functions, an input disturbance and the output response for a linear system are related simply as follows:

$$\Phi_o(f) = \Phi_i(f)\left|T(f)\right|^2 \tag{4}$$

where

Φ_i input disturbance such as the vertical gust velocity

Φ_o output response such as the airplane normal acceleration
                considered herein

$\left|T(f)\right|$ amplitude of the system frequency-response function which in
                the present application is the amplitude of the airplane
                normal-acceleration response to sinusoidal gust disturbances
                of frequency f

For a known input spectrum, equation (4) permits the determination of the
spectrum of the system response. This relation will be used in the ana-
lytical calculation of spectrums of model normal acceleration in rough
air.

Aside from determining the mean-square value of the disturbance,
the spectrum in particular cases also permits the determination of other
statistical characteristics of the disturbance that are of interest. In
particular, if the disturbance has a normal (Gaussian) probability dis-
tribution with a mean value of zero, the probability distribution is
defined by the single parameter, the mean square. In the normal distri-
bution case, the root-mean-square value provides a simple and direct
measure of the disturbance intensity. Also, S. O. Rice, in reference 5,
has derived results which in special cases permit the derivation from
the spectrum of the average number of peaks per second that exceed a
given intensity. This type of quantity is particularly useful for fatigue
studies. There is some indication as described in reference 3 that air-
plane gust loads may tend to have a normal distribution. As a consequence,
the normality of the observed distributions of intensity from the present
test results are of interest. The normality of the observed distributions
and the applicability of relations given by S. O. Rice will consequently
be examined.

Experimental Results

Characteristics of time history.- As an illustration of the general
characteristics of the normal-acceleration response for the present model
in flight through continuous rough air, two sections of record obtained
at Mach numbers of 0.83 and 1.00 are reproduced in figure 6. For com-
parison, portions of the normal-acceleration response in smooth and in
rough air of a geometrically similar model tested on the same day are
shown in figures 7(a) and 7(b), respectively. This latter model spent
only a short portion of its flight time in rough air as it reached a
maximum altitude of 6,200 feet. Figure 7(a) shows the character of the

records for smooth-air flight at a Mach number of 0.90 and an altitude of 5,700 feet. Figure 7(b) shows rough-air flight at the same Mach number but at an altitude of 2,700 feet. The smooth-air flight shows undisturbed motion whereas the rough-air flight shows disturbed motion similar to figure 6. This comparison clearly indicates that the disturbed motion of the present test is a consequence of atmospheric turbulence and not some aerodynamic phenomenon such as buffeting. The presence of smooth air above 5,000 feet is also indicated by the extrapolation of the survey-airplane data of figure 4.

Evaluation of records.- When the test data were evaluated, the time history of normal acceleration at the center of gravity was divided into sections in which the change in model forward speed was less than 5 percent. The total length of record available covered 22 seconds; however, about 8 seconds of this record length was for power-on flight and for changing model trim lift and could not be used for analysis. Eight sections of record averaging about 1.7 seconds each for various Mach numbers covering the range of 0.81 to 1.00 were obtained. The normal-acceleration data were read at intervals of 0.01 second and gave an average of about 170 data points per record section. The results of this basic-data evaluation are presented at the appropriate average Mach number for each test section and for the average altitude. Initially, these results are presented as measured; that is, no corrections for differences in turbulence intensity with altitude were made. In order to describe the characteristics of the data in detail, the following types of results were determined:

(a) Frequency distribution of measured normal-acceleration increments

(b) Power-spectral-density functions of normal acceleration

(c) Measured number of peak accelerations per mile of flight exceeding given values

The frequency distributions of normal-acceleration increment for each record section are presented in figure 8. Also shown in figure 8 for comparison are fitted normal-distribution curves which were determined from the value of the mean-square normal acceleration for each record section.

The power-spectral-density functions for normal acceleration were computed from the time-history data by using the procedures recommended in reference 6. Sixty estimates of power equally spaced over the frequency range of 0 to 50 cps were obtained for each record section. These estimates were faired and the results obtained are shown in figure 9 with the ordinate having the dimensions of g^2/cps and the abscissa given in cycles per second. Because of the characteristics of the equations used in deriving these estimates (ref. 6) each point on the curves of figure 9 represents an estimate of the average power of a frequency band width of

3.3 cps. The direct effect of this band width in the present case is a reduction in the sharpness or peak values of these spectrums, as will be discussed later.

The average number of peak-acceleration increments that exceed given levels of intensity were determined from counts of peak accelerations on the record and the results obtained are shown for each record section in figure 10. The relations derived by S. O. Rice for the case of a normal distribution permit the determination of the number of peaks that exceed given values from the power-spectral-density function. These relations were used with the spectrums of figure 9. The results obtained are also shown in figure 10 and provided a measure of the applicability of the relations derived by Rice.

The significant results of the present test are summarized in figure 11 which presents the variation of acceleration increment with Mach number as measured by the root-mean-square normal acceleration. In order to obtain the variation of root-mean-square values with Mach number, it was necessary to adjust the test results for difference in turbulence intensity at the various altitudes. The results obtained from the airplane-survey data of figure 4 were used in making these adjustments. The measured values of σ were adjusted to the intensity level at an altitude of 1,500 feet by multiplying the σ of each record section by the ratio of $\left(U_{de}\right)_{max}$ at 1,500 feet to $\left(U_{de}\right)_{max}$ at the altitude for the record section. The values of σ shown in figure 11, however, still include minor differences in weight and moment of inertia of the model as given in table I.

Analytical Calculations

In order to determine whether the observed characteristics of the load history in rough air and their variation with test condition agreed with what would be expected from theoretical considerations, the power-spectral-density functions for normal acceleration were calculated. These calculations were made for conditions (Mach number, altitude, weight) corresponding to the actual model test conditions. These calculations were made with equation (1) and depended essentially upon the estimation of the power spectrums of vertical gust velocity encountered by the model and the model frequency-response functions for a vertical gust disturbance. The methods used in estimating these functions are described in the following paragraphs.

Power spectrum of gust velocity.- The spectrums of atmospheric turbulence required are the spectrums of the gust disturbances actually experienced by the model at the various test conditions. Unfortunately, direct estimates of these spectrums were not available as the model was not equipped with any independent system of gust measurement. The

survey-airplane measurements, although useful in establishing rough over-
all averages of gust-intensity levels, do not appear adequate for this
purpose for two main reasons. First, because of the patchy nature of the
turbulence, they do not permit the accurate determination of the turbu-
lence level for the short portion of the model flight path at a given
test condition. As a second limitation, the airplane-response measure-
ments, because of differences in natural frequency, did not provide a
reliable means of determining the gust spectrum over the range of shorter
wave lengths of importance to the model. Because of these limitations,
it was necessary to estimate the turbulence spectrum by other means as
will be described in the following paragraphs.

The available measurements of the spectrum of atmospheric turbulence
in the free atmosphere have, in general, been obtained from measurements
of airplane response (see, for example, ref. 7). Although the spectrums
obtained varied in intensity, a common feature of the measurements so far
reported is the general shape of the spectrum. For the range of gust wave
lengths available, from 200 to about 2,000 feet, the spectrums were roughly
inversely proportional to the frequency squared. However, it was antici-
pated that the model accelerations would arise principally from the gust
components having wave lengths between roughly 20 and 200 feet. A need
thus existed for the determination of the shape of the spectrum of gust
velocity over this range of shorter wave lengths.

Inasmuch as measurements of airplane responses do not readily per-
mit the determination of the gust spectrum at the smaller wave lengths
of interest, recourse was made to some available measurements of high-
frequency airspeed fluctuations obtained in flight through rough air.
These measurements were obtained in flight at about 400 feet above ter-
rain with a liaison-type airplane. The airspeed-measuring system had been
specially designed for these tests to yield a flat response to disturbances
having frequencies up to 10 cps. (For the flight speed of this airplane,
this response corresponds to wave lengths of roughly 13 feet.) Since the
airplane can be reasonably considered to be longitudinally insensitive to
horizontal-gust disturbances of high frequency, it appeared reasonable to
assume that the high-frequency airspeed fluctuations were a direct reflec-
tion of the horizontal-gust fluctuations. A determination of the spectrum
of these fluctuations indicated that the spectrum could be approximated

by the expression $0.052/\Omega^2$ for the wave lengths of 13 feet to several
hundred feet. It is also of interest to note that some measurements of
the spectrum of vertical gust velocity recently obtained by the NACA by
using an angle-of-attack indicator confirmed this shape for the range of
wave lengths being considered. In view of these considerations, it
appeared reasonable to assume that the shape of the spectrum of vertical
turbulence encountered by the model could be approximated by the relation

$$\Phi_i(\Omega) = K/\Omega^2 \qquad\qquad (0.003 < \Omega < 0.5)$$

where K is a constant and is a measure of the turbulence intensity.

The value of K appropriate to the model test conditions was esti-
mated from simple considerations of the relative gust experience of the
survey airplane for the model tests and the airplane used to obtain the
shape of the gust spectrum. Evaluations indicated that the gust intensi-
ties for the model test conditions at 1,500 feet were roughly 60 percent
as severe as those obtained from flights to determine the shape of the
gust spectrum. Since the spectrum is a function of the gust velocity
squared, the appropriate value for K would be $0.052 \times (0.6)^2$ or
roughly 0.018. Thus, the spectrum of gust velocity assumed for the model
test conditions was

$$\Phi_i(\Omega) = 0.018/\Omega^2$$

This spectrum was assumed to apply to the model flight at the 1,500-foot-
altitude level.

Frequency-response function.- The analytical determination of the
frequency-response function for the present test conditions is a diffi-
cult problem because of the rapid changes in the aerodynamic character-
istics in the transonic speed range and the lack of information on the
unsteady aerodynamic forces for gust disturbances in this region. As a
preliminary effort, simple estimates were used in order to determine
whether the spectrum of the measured load responses and their variations
with test condition agreed with what would be expected.

The procedure assumed that the model in penetrating a sharp-edged
gust behaves essentially as a one-degree-of-freedom system (vertical
motion only) up to the first peak acceleration. For the present values
of mass parameter μ_g (150), the first peak may be expected to occur at
about 12 chords of gust penetration. The response up to 12 chords was
estimated from the results of reference 8. Beyond 12 chords, it was
assumed that the model was free of the transient gust disturbance and
responded essentially in short-period longitudinal motion. From these
considerations, the model response to a step gust was taken to be repre-
sented by the following relations:

$$\Delta n(x) = \Delta n_{max} \sin \frac{\pi}{24\bar{c}} x \qquad\qquad (0 \leqq x \leqq 12\bar{c})$$

$$\Delta n(x) = \Delta n_{max} e^{-b(x-12\bar{c})} \cos \Omega_o (x - 12\bar{c}) \qquad (12\bar{c} \leqq x \leqq \infty)$$

where Δn_{max} is the first peak acceleration,

$$\bar{c} = 0.821 \text{ foot}$$

$$b = \frac{0.693}{T_{1/2}V}$$

$$\Omega_0 = \frac{2\pi}{VP}$$

The model short-period characteristics $T_{1/2}$ and P were obtained from stability tests summarized in figure 3. Although this procedure for estimating the step response is obviously crude, it was nevertheless believed to reflect the essential variations of the load response over the limited range of test conditions.

The responses to the step gust obtained in the manner outlined were in turn used to estimate the frequency-response function by the relation:

$$T(\Omega) = i\Omega \int_0^\infty \Delta n(x)e^{-i\Omega x}dx$$

The integration was completed in closed form and the amplitude squared of the frequency-response functions obtained (fig. 12) for the various test conditions. The results are shown separately for the two weight conditions.

Normal-acceleration spectrums.- From the calculated frequency-response functions and the gust spectrum previously derived, the spectrums of acceleration increment were obtained by application of equation (1). The results obtained are shown in figure 13 for the eight test conditions. The root-mean-square values σ were obtained from these spectrums through use of equation (3). These root-mean-square values were used to obtain the curves shown in figure 11, which represent the expected variation of the root-mean-square value with Mach number for the two weight conditions corresponding to before and after sustainer rocket burning.

Inasmuch as the measured spectrums of figure 9 represent an averaged spectrum over a frequency band width of roughly 3.3 cycles per second, the spectrums of figure 9 are not directly comparable with those calculated (fig. 13). In order to obtain calculated results that are directly comparable with those measured, these calculated spectrums of figure 13

were modified by the use of an averaging function roughly equivalent to the one present in the measured spectrums. Specifically, a triangular weighting function having a base band width of 3.3 cps was used. The modified spectrums are shown in figure 14(a) and apply to the turbulence intensity of the 1,500-foot-altitude level. The measured spectrums of figure 9 are for different altitudes and their corresponding turbulence intensities. In order to make the measured spectrums of figure 9 directly comparable with the modified calculations of figure 14(a), the measured

spectrums of figure 9 were multiplied by the ratio $\left[\dfrac{(U_{de})_{1,500 \text{ feet}}}{(U_{de})_{\text{test altitude}}}\right]^2$

and the results are shown in figure 14(b). In order to permit detailed comparison between these measured and calculated spectrums, the abscissa scale of figure 14 has been amplified.

DISCUSSION

Characteristics of model rough-air response.- Examination of the missile normal accelerations in rough air, figures 6(a) and 6(b) indicates that in rough air the model undergoes a sustained and irregular oscillation with an apparent predominant frequency at about the airplane short-period frequency. For the two cases shown in the figure, the predominating frequency appears at roughly 12 cps at Mach number 1.0 and about 10 cps at Mach number 0.83. These frequencies agree closely with the model short-period frequency as indicated by figure 3. That this disturbed motion is a consequence of atmospheric turbulence and not of some aerodynamic phenomenon such as buffeting has been indicated by figure 7 in which the normal acceleration of a similar model at Mach number 0.9 is shown in smooth and rough air. In smooth air at the higher altitude, the model apparently shows little disturbed motion; whereas, at the same Mach number and in rough air at lower altitudes, the model exhibits the same characteristic type of disturbed motion described in the test-model measurements of figure 6.

Frequency distributions of acceleration increments.- The frequency distributions of the acceleration increments for the eight test sections of record shown in figure 8 exhibit in almost all cases a unimodal character and appear roughly symmetrical with a mean of zero. Considering the small amount of data available in each case, the observed distributions appear to be approximated fairly well by the fitted normal-distribution curves shown. It thus appears reasonable from these results to assume that the distribution of the acceleration increment is roughly normal and, as a consequence, the root-mean-square value provides a direct measure of the response intensity. Although some differences in turbulence intensity associated with the differences in altitude exist for the various test

conditions of figure 8, the distributions show increases in spread as
the Mach number increases. At the higher Mach numbers relatively more
observations are noted at the higher values of acceleration increment;
thus, an increase in the acceleration level with Mach number was indicated.

Peak accelerations.- The comparison of the peak accelerations obtained
by a count from the time-history records with those calculated from the
measured spectrums using the results of reference 5, in general, shows rela-
tively good agreement for the cases of figure 10. However, in some cases,
the results obtained by counting peaks on the record show sizeable depar-
ture from those calculated from the measured spectrums for the larger values
of Δn. This effect is possibly due to the small record length at each
test condition. In general, the results imply that the relations between
the spectrums and the peak counts derived by Rice in reference 5 appear
applicable and yield good approximations for the test conditions
investigated.

Power spectrums of acceleration increments.- Consideration of the
measured spectrums of acceleration increment of figure 9 indicates the
narrow-band-pass filter characteristics of the model response. These
measurements, as previously mentioned, actually represent averages over
frequency band widths of 3.3 cps. Thus the actual spectrums are even
more peaked. In each case, the spectrum is largely concentrated between
6 and 12 cps with some indication of a small secondary peak at 35 cps or
the fundamental wing-bending mode. For both weight conditions, the spec-
trums, in general, exhibit a progressive movement to higher frequency as
the Mach numbers increase; this movement reflects the increase in the
short-period frequency with Mach number. In addition, these spectrums
also show a general tendency for increasing peakedness with increasing
Mach number and reflect the deterioration of the short-period damping
over this Mach number range.

If the measured spectrums of figure 9 are compared with the calculated
ones in figure 13, it will be noted that a good correlation exists between
the distribution of the power between the sets of spectrums. In both cases,
the peaks occur close to the short-period frequencies. The measured
spectrums, however, are not as peaked as those calculated, the peak ampli-
tudes being in each case roughly from one-half to one-third as large.
This difference, as previously mentioned, is in large part a consequence
of the band-width characteristic of the measured estimates. When the
calculated spectrums are modified to incorporate the same type of averages
over a band width of 3.33 cps and the measured spectrums adjusted for
turbulence-intensity variations, better agreement is obtained. However,
if the calculated and measured spectrums are compared in detail (fig. 14),
some differences are still present. These discrepancies arise from a num-
ber of sources. The principal causes appear to be associated with the fol-
lowing: rather large statistical fluctuations associated with the rela-
tively small samples (roughly 2 seconds in each case), the inaccuracy of

the estimates of the turbulence intensity obtained from the airplane sur-
vey data of the turbulence intensity for the short sections of record at
each test condition, and the crude analysis used in estimating the air-
plane frequency-response function. Since the first two sources of dis-
crepancy can in each case contribute variations of the order of ±20 per-
cent, the observed discrepancies appear to be within the precision of the
techniques used.

   Variation of acceleration with Mach number.- The significant results
concerning the variation of acceleration with Mach number are summarized
in figure 11 which compares the measured and calculated root-mean-square
accelerations as a function of Mach number. In general, the experimental
data appear to follow the trends of the calculated curves. The root-mean-
square acceleration appears to increase from roughly 0.35g at a Mach num-
ber of 0.8 to 0.65g at a Mach number of 0.95, an increase of roughly
85 percent. Of this 85-percent increase, roughly only 30 percent appears
attributable to direct effects associated with speed increase and with
the change in slope of the lift curve for this Mach number range. The
remaining 55 percent appears to be essentially a reflection of the dete-
rioration of damping over this Mach number range.

   It is of interest to note that two test values were obtained at both
Mach number 0.88 and 0.91 (fig. 11). Although the two test values at about
M = 0.91 are in good agreement, a difference of roughly 30 percent will
be noted in the two values obtained at Mach number 0.88. Also, the experi-
mental value for Mach number 0.81 appears inconsistent with the remaining
experimental results and roughly 30 percent higher than might be expected.
These results illustrate scatter previously noted in the more detailed
discussions of spectrums and essentially reflect a limited precision in
the technique used in the test.

   Evaluation of technique.- The foregoing results indicate the applica-
bility and limitations of the rocket-powered-model technique for gust-loads
studies. Turbulence conditions suitable for testing rocket-powered models
in rough air appear to exist over the model firing range under some weather
conditions. These conditions can be predicted in advance for the purpose
of test planning. The airplane survey flights provided a rough overall
measure of the level of turbulence experienced by the model. The results
of the first test show that the technique at present is suitable for the
study of large-order effects. In detail, the results (for example,
fig. 11) show scatter. It thus appears that, although the present tech-
nique is suitable for the study of trends, it is not yet precise enough
for the study of small-order effects.

   In the study of many gust-loads problems, greater precision than
obtained in the present test results is frequently desirable. The pri-
mary sources limiting the precision in the present test appear to be
associated with the patchy character of the turbulence, the inability of
the airplane survey data to describe precisely the level of the turbulence

experienced by the model, and the statistical fluctuations introduced by the short sampling length at each test condition. An increase in precision necessary for making detailed studies can probably be obtained by the following modifications in the technique:

(a) Greater selectivity in the choice of test conditions to obtain more homogeneous turbulence

(b) Equip the model with an independent system for measuring the turbulence such as a sensitive high-frequency airspeed-fluctuation measuring system

(c) Obtain longer record samples by equipping the model with a slow-burning sustainer rocket or by decreasing the rate of deceleration.

CONCLUDING REMARKS

The investigation of the application of rocket models for the study of gust loads at high speeds has yielded information on the feasibility and precision of the technique, the behavior of a tailless rocket-powered model in rough air from Mach number 0.80 to 1.00, and the applicability of power-spectral methods of analysis to measurements of this type. The results obtained on each of these subjects are noted in the following paragraphs.

A technique for testing rocket-powered models in rough air was evolved and incorporated the use of a survey airplane. This technique in its present form is feasible and practical for the study of large-order effects on gust loads. The limited precision obtained in the first test indicates a need for refinements in order to permit study of small-order effects. Several refinements to the technique are suggested for increasing the precision.

The tailless rocket-powered model tested showed the adverse effects of low damping in pitch on the normal-acceleration response in rough air. The model short-period frequency was dominant in the normal-acceleration response. The load intensity as measured by the root-mean-square normal acceleration showed a rapid increase of roughly 85 percent as the Mach number increased from 0.80 to 1.00. Of this 85 percent, about 55 percent appeared to be associated with the rapid decrease in pitch damping over this Mach number range. These results were in substantial agreement with theoretical calculations presented.

Power-spectral methods were used both in the evaluation of the test results and in the associated theoretical calculations and proved a useful means for both describing the test results and correlating them with the

theoretical calculations. The measured probability distributions appeared to approximate the normal (Gaussian) distribution curve. The relations derived by S. O. Rice between the spectrum and the number of peak accelerations per second were applied to the measured spectrums and yielded satisfactory approximations to the observed results. These two results, the normality of the observed accelerations and the apparent applicability of Rice's results, appear to make the power-spectral methods of analysis particularly useful.

Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., November 23, 1953.

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7. Clementson, Gerhardt C.: An Investigation of the Power Spectral Density of Atmospheric Turbulence. Ph. D. Thesis, M.I.T., 1950.

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TABLE I.- MODEL CHARACTERISTICS

Wing Dimensions:
   Wing area (including fuselage), sq ft 3.88
   Mean aerodynamic chord, ft 0.821
   Aspect ratio 6.0
   Taper ratio 0.60
   Airfoil section . . . . NACA 65A009

Mass Characteristics:

	Before sustainer rocket burning	After sustainer rocket burning
Weight, lb	85.3	75.0
Center-of-gravity location, percent M.A.C. (ahead of L.E.)	-77.5	-77.8
I_y, slug-ft^2	5.10	4.80
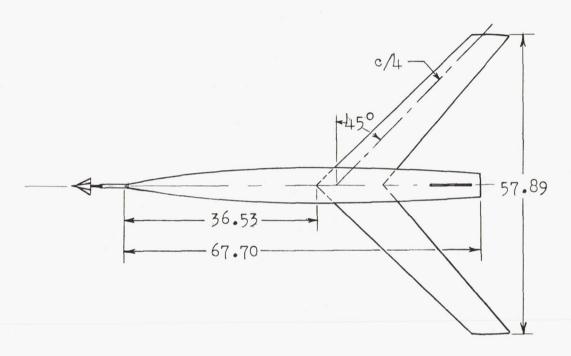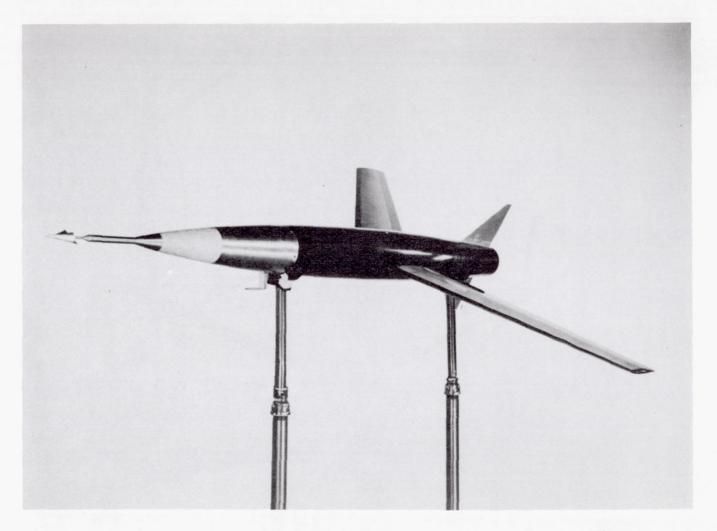
Figure 1.- Physical characteristics of model (all dimensions in inches).

L-76425.1

(a) Three-quarter front view.

Figure 2.- Photographs of model.

(b) Model on launcher.

Figure 2.- Concluded.

L-77505.1
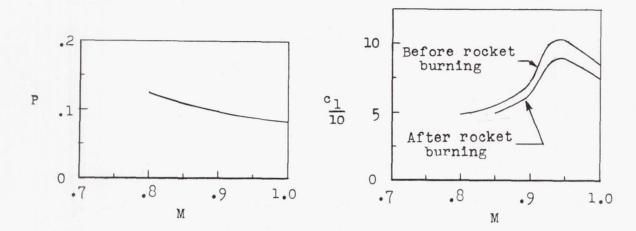
Figure 3.- Model stability characteristics.
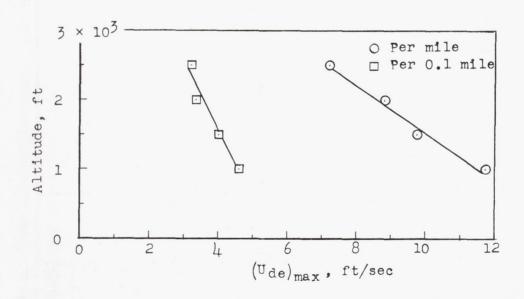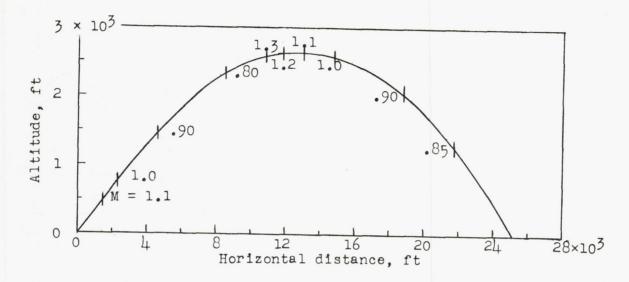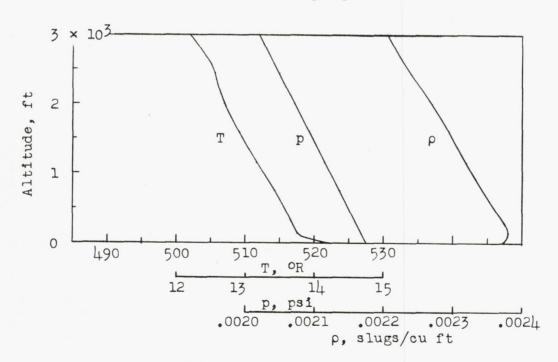
Figure 4.- Variation of gust intensity with altitude as indicated by the
survey airplane.
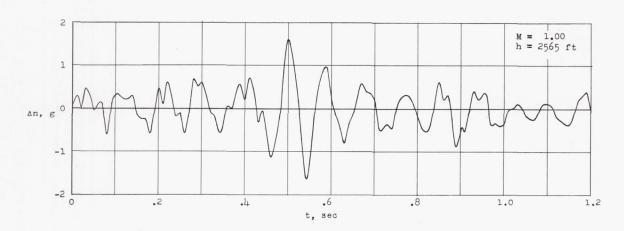
(a) Model flight path.
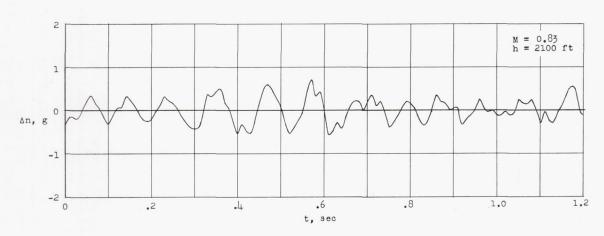
(b) Properties of atmosphere.

Figure 5.- Model flight conditions.

(a) Experimental, M = 1.00.
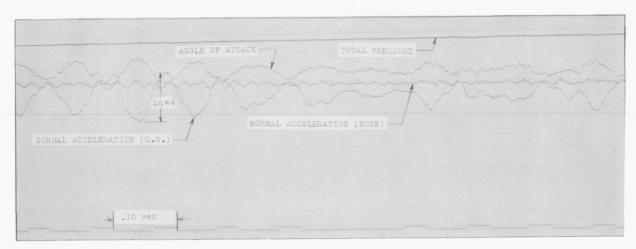
(b) Experimental, M = 0.83.

Figure 6.- Time histories of normal acceleration.

(a) Smooth air at M = 0.90; altitude, 5,700 feet.

(b) Rough air at M = 0.90; altitude, 2,700 feet.

Figure 7.- Comparison of smooth- and rough-air portions of flight.

(a) Model weight, 85.3 lb; I_y = 5.1 slug-ft^2.
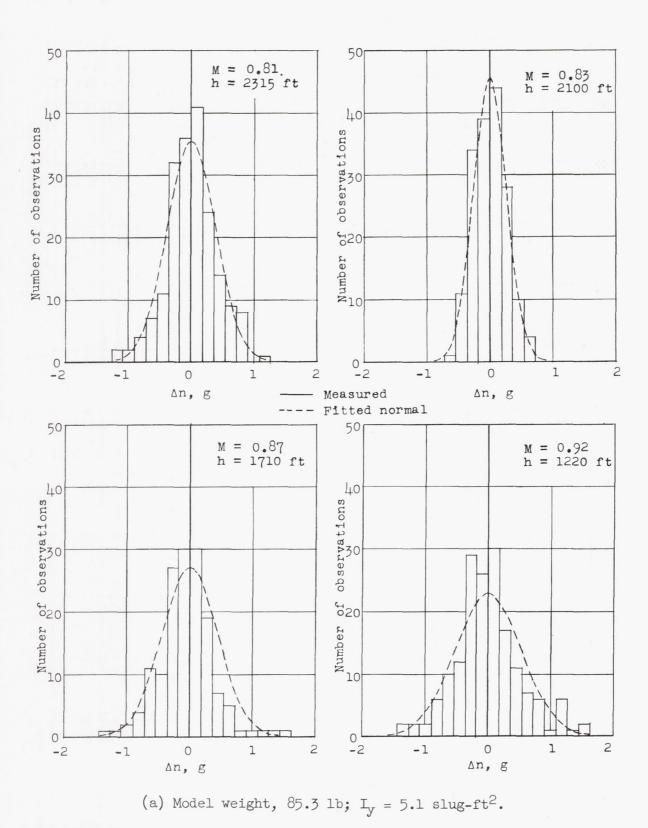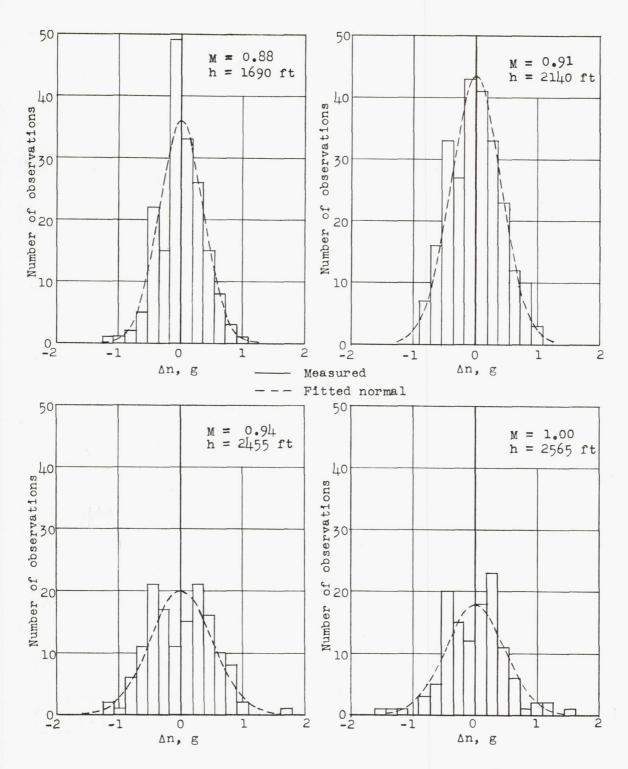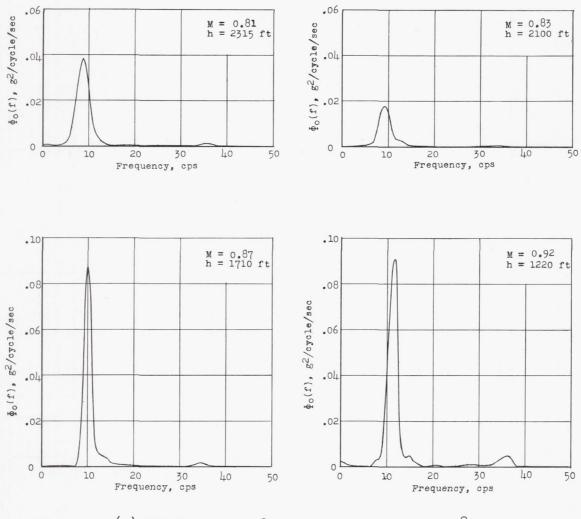
Figure 8.- Comparison of observed frequency distribution and fitted normal
frequency distribution.
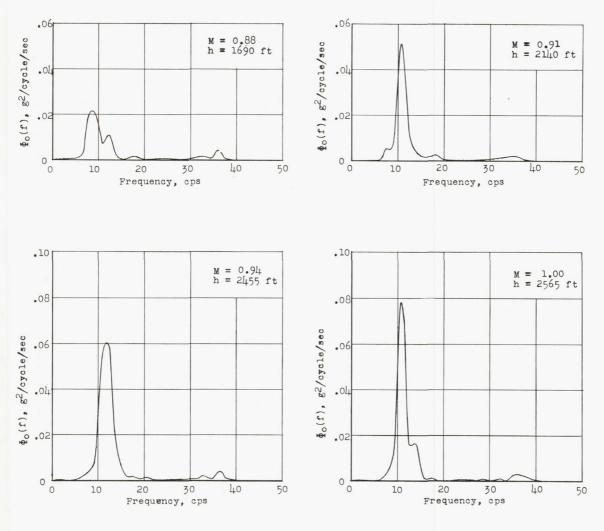
(b) Model weight, 75 lb; $I_y = 4.8$ slug-ft^2.

Figure 8.- Concluded.
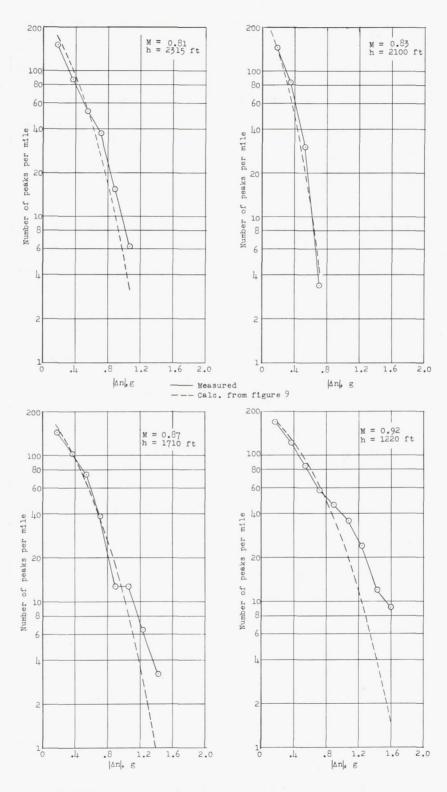
(a) Model weight, 85.3 lb; I_y = 5.1 slug-ft^2.

Figure 9.- Power-spectral-density functions of experimental normal-acceleration data.
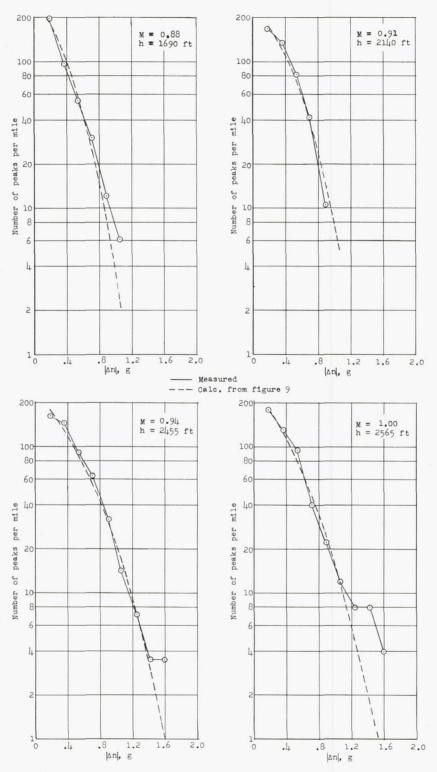
(b) Model weight, 75 lb; $I_y = 4.8$ slug-ft^2.

Figure 9.- Concluded.

(a) Model weight, 85.3 lb; $I_y = 5.1$ slug-ft^2.

Figure 10.- Distribution of peak normal-acceleration increments.

(b) Model weight, 75 lb; I_y = 4.8 slug-ft^2.

Figure 10.- Concluded.
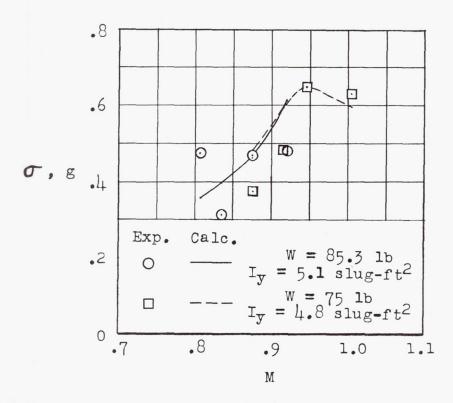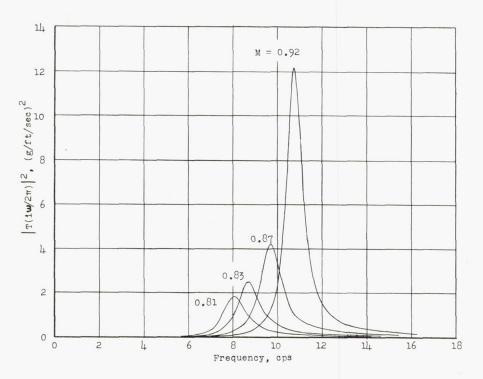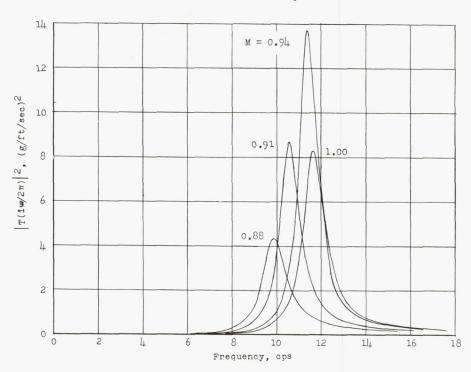
Figure 11.- Variation of experimental and calculated root-mean-square normal acceleration with Mach number.
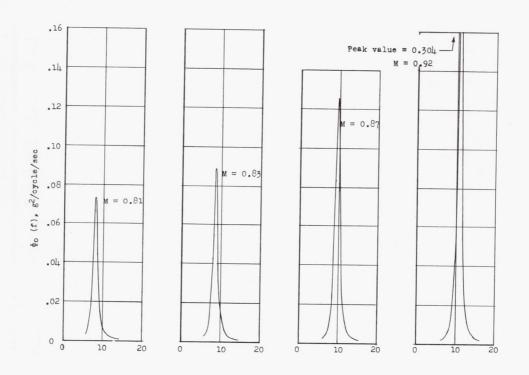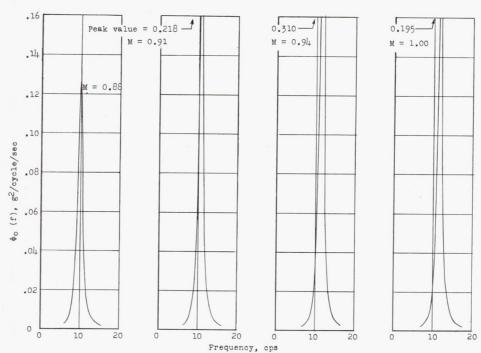
(a) Model weight, 85.3 lb; $I_y = 5.1$ slug-ft^2.

(b) Model weight, 75 lb; $I_y = 4.8$ slug-ft^2.

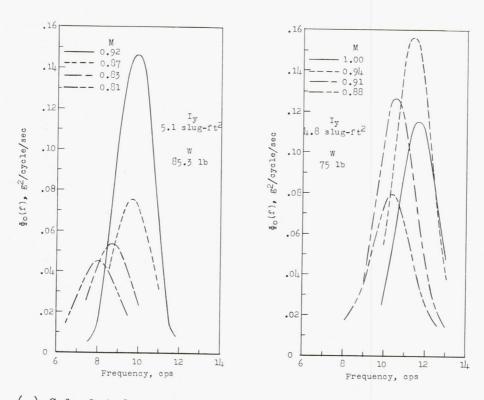Figure 12.- Amplitude squared of the model transfer functions for several test conditions.

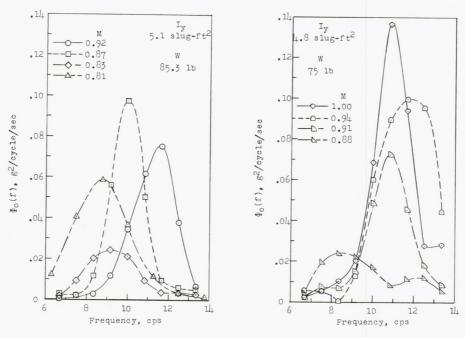(a) Model weight, 85.3 lb; I_y = 5.1 slug-ft^2; altitude, 1,500 ft.

(b) Model weight, 75 lb; I_y = 4.8 slug-ft^2; altitude, 1,500 ft.

Figure 13.- Calculated power-spectral-density functions of normal acceleration.

(a) Calculated spectrums modified with a 3.3-cps filter.

(b) Experimental spectrums.

Figure 14.- Power-spectral-density functions of normal acceleration for the turbulence intensity at 1,500 feet altitude.